රුවන් දර්ශන

B.Sc. (Hons)

සාධාරණ පද ලිවීම.

01. $1.4.7 + 4.7.10 + 7.10.13 + \dots\dots$

$U_r = [1 + (r-1)3]\ [4 + (r-1)3]\ [7 + (r-1)3]$

$U_r = (3r-2)(3r+1)(3r+4)$

02. $2.7.12.17 + 7.12.17.22 + 12.17.22.27 + \dots\dots$

$U_r = [2 + (r-1)5]\ [7 + (r-1)5]\ [12 + (r-1)5]\ [17 + (r-1)5]$

$U_r = (5r-3)(5r+2)(5r+7)(5r+12)$

03. $1.2.3 + 2.3.4 + 3.4.5 + \dots\dots$

$U_r = r(r+1)(r+2)$

04. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots\dots$

$U_r = \frac{1}{r(r+1)}$

05. $\log 1^3 + \log 3^3 + \log 5^3 + \dots\dots$

$U_r = \log [1 + (r-1)2]^3$

$U_r = \log (2r-1)^3$

06. $\frac{1}{1.2} \cdot \frac{1}{3} + \frac{1}{2.3} \cdot \frac{1}{3^2} + \frac{1}{3.4} \cdot \frac{1}{3^3} + \dots\dots$

$U_r = \frac{1}{r(r+1)} \cdot \frac{1}{3^r}$

07. $1.1 + 2.2 + 3.3 + \dots\dots + n.n!$

$U_r = r.r!$

08. $1 + (1+2) + (1+2+3) + (1+2+3+4) + \dots\dots$

$U_r = 1 + 2 + 3 + \dots\dots + r$

$U_r = \frac{r}{2}(r+1)$

09. $\frac{1}{1^2} + \frac{1}{1^2 + 2^2} + \frac{1}{1^2 + 2^2 + 3^2} + \dots\dots$

$U_r = \frac{1}{1^2 + 2^2 + 3^2 + \dots + r^2}$

$= \frac{1}{\frac{r}{6}(r+1)(2r+1)} = \frac{6}{r(r+1)(2r+1)}$

10. ශ්‍රේණියක පද n වල ඓක්‍යය $(n+1)^3$ වේ. r වෙනි පදය සොයන්න.

$$U_n = S_n - S_{n-1}$$
$$= (n+1)^3 - n^3$$
$$U_n = 3n^2 + 3n + 1$$
$$U_r = 3r^2 + 3r + 1$$

විසඳු ගැටළු 01

පළමු පද n වල ඓක්‍යය සොයන්න.

Σ සිද්ධාන්තය

11. $1.3 + 3.5 + 5.7 + \ldots\ldots\ldots$

$$U_r = (2r-1)(2r+1)$$
$$U_r = 4r^2 - 1$$
$$\sum_{r=1}^{n} U_r = \sum_{r=1}^{n} (4r^2 - 1)$$

$$S_n = 4\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} 1$$
$$= 4 \cdot \frac{n}{6}(n+1)(2n+1) - n \cdot 1$$
$$= \frac{n}{3}[2(n+1)(2n+1) - 3]$$

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12. $1.2.3 + 2.3.4 + 3.4.5 + \ldots\ldots\ldots$

$$U_r = r(r+1)(r+2)$$
$$\sum_{r=1}^{n} U_r = \sum_{r=1}^{n} r(r+1)(r+2)$$
$$= \sum_{r=1}^{n} r^3 + 3r^2 + 2r$$
$$= \sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2 + z\sum_{r=1}^{n} r$$
$$= \left[\frac{n}{2}(n+1)\right]^2 + 3 \cdot \frac{n}{6}(n+1)(2n+1) - 2 \cdot \frac{n}{2}(n+1)$$
$$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{3} - n(n+1)$$

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13. $1.2^2 + 3.3^2 + 5.4^2 + \ldots\ldots\ldots$ පද n වල ඓක්‍යය

$$U_r = (2r-1)(r+1)^2$$
$$\Sigma U_r = \Sigma(2r-1)(r+1)^2$$
$$= \Sigma(2r-1)(r^2+2r+1)$$
$$= \Sigma(2r^3 + 3r^2 - 1)$$
$$= 2\Sigma r^3 + 3\Sigma r^2 - \Sigma 1$$
$$= 2\left[\frac{n}{2}(n+1)^2 + \frac{3.n}{6}(n+1)(2n+1) - n\right.$$

14. $1 + (1+2) + (1+2+3) + (1+2+3+4) + \dots\dots\dots$

$U_r = 1 + 2 + 3 + \dots\dots\dots + r$

$U_r = \frac{r}{2}(r+1)$

$\sum_{r=1}^{n} U_r = \sum_{r=1}^{n} \frac{r}{2}(r+1)$

$Sn = \frac{1}{2}\sum_{r=1}^{n}(r^2+1)$

$= \frac{1}{2}\left[\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right]$

$= \frac{1}{2}\left[\frac{n}{6}(n+1)(2n+1) + \frac{n}{2}(n+1)\right]$ //

අන්තර් සිද්ධාන්තය :-

15. $1.2.3 + 2.3.4 + 3.4.5 + \dots\dots\dots$

$U_r = r(r+1)(r+2)$

$f(r) = \frac{r(r+1)(r+2)(r+3)}{4 \times 1}$

$f(r) - f(r-1) = U_r$ වේ.

දැන් අන්තර් සිද්ධාන්තය යොදමු.

$U_r = f(r) - f(r-1)$

$r = 1 \quad U_1 = f(1) - f(0)$

$r = 2 \quad U_2 = f(2) - f(1)$

$r = 3 \quad U_3 = f(3) - f(2)$

$r = n-1 \quad U_{n-1} = f(n-1) - f(n-2)$

$r = n \quad U_n = f(n) - (n-1)$

$(+)$

$\sum_{r=1}^{n} U_r = f(n) - f(0)$

$r = 1 \quad = \frac{n(n+1)(n+2)(n+3)}{4}$ //

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16. $1.6.11 + 6.11.16 + 11.16.21 + \dots\dots\dots$ පද n වල ඓක්‍යය

1 ක්‍රමය

$U_r = [1 + (r-1)5][6 + (r-1)5][11 + (r-1)5]$

$U_r = (5r-4)(5r+1)(5r+6)$

$f(r)^2 = \frac{(5r-4)(5r+1)(5r+6)(5r+11)}{4 \times 5}$

$f(r) - f(r-1) = U_r$ වේ.

දැන් අන්තර් සිද්ධාන්තය යොදමු.

$$U_r = f(r) - f(r-1)$$

$r = 1 \quad U_1 = f(1) - f(0)$

$r = 2 \quad U_2 = f(2) - f(1)$

$r = 3 \quad U_3 = f(3) - f(2)$

$r = 1 \quad U_{n-1} = f(n-1) - f(n-2)$

$r = n \quad U_n = f(n) - (n-1)$

$$\Sigma U_r = f(n) - f(c)$$

$$= \frac{(5n-4)(5n+1)(5n+6)(5n+11)}{20}$$

$$= \frac{(-4).1.6.11}{20}$$

2 ක්‍රමය

$U_r = (5r-4)(5r+1)(5r+6)$

$f(r) = (5r-4)(5r+1)(5r+6)(5r+11)A$ ලෙස ගනිමු.

$f(r) - f(r-1) = U_r$ වන පරිදි A සොයමු.

$[(5r-4)(5r+1)(5r+6)(5r+11)A] - [(5r-9)(5r-4)(5r+1)(5r+6)]A$

$= (5r-4)(5r+1)(5r+6)$

$A - [(5r-9)B](5r+1)$

විසඳූ ගැටළු 02

ගැටළු

17. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots\ldots$ පද n වල ඓක්‍යය

18. $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots\ldots$ පද n වල ඓක්‍යය

19. $\frac{4}{1.2.3} + \frac{5}{2.3.4} + \frac{6}{3.4.5} + \ldots\ldots\ldots$ ශ්‍රේණියේ $U_r = \frac{r+3}{r(r+1)(r+2)}$ ලෙස දී ඇත.

$U_r = \frac{3}{2}[f(r) - f(r+1)] + \frac{1}{2}[f(r+2) - f(r+1)]$ වන පරිදි $f(r)$ ශ්‍රිතය සොයන්න.

එනයින් $\sum_{r=1}^{n} U_r = \frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)}$ බව පෙන්වන්න.

20. $\frac{2}{3.4.5} + \frac{3}{4.5.6} + \frac{4}{5.6.7} + \frac{5}{6.7.8} + ...$ ශ්‍රේණියේ r වන පදය U_r ලියන්න.

$U_r = f(r) - f(r-1)$ වන පරිදි $f(r)$ සොයන්න. එමගින් $\sum_{r=1}^{n} U_r$ සොයන්න.

$\sum_{r=1}^{\infty} U_r \; \frac{5}{24} =$ බව අපෝහනය කරන්න. $\sum_{r=1}^{\infty} U_r$ ශ්‍රේණිය අභිසාරී ද, පිළිතුර සනාථ කරන්න.

21. $\frac{2(r+3)}{2(r+1)(r+2)} = \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$ වන පරිදි A හා B නියත සොයන්න.

$\frac{8}{1.2.3}\left(\frac{1}{3}\right)^1 + \frac{10}{2.3.4}\left(\frac{1}{3}\right)^2 + \frac{12}{3.4.5}\left(\frac{1}{3}\right)^3 + ...$ ශ්‍රේණියේ r වන පදය U_r ලියා දක්වන්න. ඉහත ප්‍රතිඵලය භාවිතයෙන් හෝ අන් ක්‍රමයකින් හෝ $\sum_{r=1}^{n} U_r$ සොයන්න.

22. $\frac{r+4}{r(r+1)(r+2)} \equiv \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$ වන පරිදි A හා B නියත සොයන්න.

$\sum_{r=1}^{n} \frac{r+4}{2^r (r)(r+1)(r+2)}$ අගයන්න. එම ඓක්‍යය S_n නම්, $0 < S_n < \frac{1}{2}$ බව පෙන්වන්න.

23. $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + ...$ ලෙස වූ පරිමිත ශ්‍රේණියේ r වෙනි පදය වන U_r ලියා දක්වන්න. r හි ශ්‍රිතයක් වන $f(r)$ ඇසුරින් $U_r = \frac{1}{2}[f(r) - f(r+1)]$ ලෙස ලිවිය හැකි බව පෙන්වන්න. එමගින් දී ඇති ශ්‍රේණියේ මුල් පද n හි ඓක්‍යය, $\frac{n(n+1)}{2(1+n+n^2)}$ බව සාධනය කරන්න.

පිළිතුරු

17. $U_r = \frac{1}{r(r+1)}$

$U_r = \frac{1}{r} \; \frac{1}{(r+1)}$

$U_r = f(r) - f(r+1)$ ආකාරය

$U_r = f(r) - f(r+1)$

$r=1 \quad U_1 = f(1) - f(2)$

$r=2 \quad U_2 = f(2) - f(3)$

$r=3 \quad U_3 = f(3) - f(4)$

$r=n \quad U_n = f(n) - (n+1)$

$\sum_{r=1}^{n} U_r = f(1) - (n+1)$

$\frac{1}{1} - \frac{1}{(n+1)} = \frac{n}{n+1}$

18. $U_r = \dfrac{1}{(2r-1)(2r+1)}$

$U_r = \dfrac{1/2}{(2r-1)} + \dfrac{-1/2}{(2r+1)}$

$U_r = \dfrac{1/2}{(2r-1)} - \dfrac{1/2}{(2r+1)}$

$U_r = f(r) - f(r+1)$ ආකාරය

$r=1 \quad U_1 = f(1) - f(2)$

$r=2 \quad U_2 = f(2) - f(3)$

$r=n \quad U_n = f(n) - f(n+1)$

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$$\sum_{r=1}^{n} U_r = f(1) - (n+1)$$

$$\frac{1/2}{(2n-1)} = \frac{1/2}{(2n+1)}$$

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19. $f(r) = \dfrac{A}{r}$

$$\frac{(r+3)}{2(r+1)(r+2)} = \frac{3}{2}\left[\frac{A}{r} - \frac{A}{r+1}\right] + \frac{1}{2}\left[\frac{A}{r+2} - \frac{A}{r+1}\right]$$

$2(r+3) = 3A(r+2)(r+1) - r + Ar[r+1-(r+2)]$

$2r+6 = A[3r+6-r]$

$2r+6 = A(2r+6) \qquad A = 1$

$\therefore f(r) = \dfrac{1}{r}$

$U_r = \dfrac{3}{2}[f(r) - f(r+1)] + \dfrac{1}{2}[f(r+2) - f(r+1)]$

$r = 1 \quad U_1 = \dfrac{3}{2}[f(1) - f(2)] + \dfrac{1}{2}[f(3) - f(2)]$

$r = 2 \quad U_2 = \dfrac{3}{2}[f(2) - f(3)] + \dfrac{1}{2}[f(4) - f(3)]$ ⋮

$r = n-1 \quad U_{n-1} = \dfrac{3}{2}[f(n-1)] - f(n)] + \dfrac{1}{2}[f(n+1) - f(n)]$

$r = n \quad U_n = \dfrac{3}{2}[f(n) - f(n+1)] + \dfrac{1}{2}[f(n+2) - f(n+1)]$

$\sum_{r=1}^{n} U_r = \dfrac{3}{2}[f(1) - f(n+1)] + \dfrac{1}{2}[f(n+2) - f(2)]$

$= \dfrac{3}{2}\left[\dfrac{1}{1} - \dfrac{1}{n+1}\right] + \dfrac{1}{2}\left[\dfrac{1}{n+2} - \dfrac{1}{2}\right]$

$= \dfrac{3}{2} - \dfrac{3}{2(n+1)} + \dfrac{1}{2(n+2)} - \dfrac{1}{4}$

$= \dfrac{6-1}{4} + \dfrac{n+1-3(n+2)}{2(n+1)(n+2)}$

$= \dfrac{5}{4} + \dfrac{-2n-5}{2(n+1)(n+2)} \qquad = \dfrac{5}{4} + \dfrac{(2n+5)}{2(n+1)(n+2)}$

20. $U_r = \dfrac{r+1}{(r+2)(r+3)(r+4)}$

$f(r) = \dfrac{Ar+B}{(r+3)(r+4)}$

$f(r-1) = \dfrac{A(r-1)+B}{(r+2)(r+3)}$

$U_r = f(r) - f(r-1)$

$\dfrac{r+1}{(r+2)(r+3)(r+4)} = \dfrac{Ar+B}{(r+3)(r+4)} - \dfrac{A(r-1)+B}{(r+2)(r+3)}$

$(r+1) = (Ar+B)(r+2) - \{A(r-1) + B(r+4)\}$

$r+1 = r\{2A+B-4A-(B-A)\} + 2B - 4(B-A)$

$r+1 = r(-A) + 4A - 2B$

$r ; 1 = -A \qquad A = -1$ //

හි : $1 = 4A - 2B$

$2B = -4 - 1$

$B = \dfrac{-5}{2}$ //

$f(r) = \dfrac{-r - \frac{5}{2}}{(r+3)(r+4)}$

$r = 1 ; \quad U_1 = f(1) - f(0)$

$r = 2 ; \quad U_2 = f(2) - f(1) \vdots$

$r = n-1 ; \quad U_{n-1}\ f(n-1) - f(n-2)$

$r = n ; \quad U_n = f(n) - f(n-1)$

$\sum_{r=1}^{n} U_r = f(n) - f(0)$

$= \dfrac{-n - \frac{5}{2}}{(n+3)(n+4)} - \dfrac{\left(\frac{-5}{2}\right)}{3 \times 4}$

$= \dfrac{5}{24} - \dfrac{\left(n + \frac{5}{2}\right)}{(n+3)(n+4)}$

$= \lim_{n \to \infty} \sum_{r=1}^{n} U_r = \lim_{n \to \infty} \dfrac{5}{24} - \dfrac{\left(n + \frac{5}{2}\right)}{(n+3)(n+4)}$

$\sum_{r=1}^{\infty} U_r = \lim_{n \to \infty} \dfrac{5}{24} - \dfrac{\left(\frac{1}{n} + \frac{5}{2n^2}\right)}{\left(1 + \frac{3}{n}\right)\left(1 + \frac{4}{n}\right)}$

$= \dfrac{5}{24}$ //

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21. $\dfrac{2(r+3)}{2(r+1)(r+2)} = \dfrac{A}{r(r+1)} + \dfrac{B}{(r+1)(r+2)}$

$2(r+3) = A(r+2) + Br$

$r = 0 \Rightarrow 6 = 2A \Rightarrow A = 3$

$r = -2 \Rightarrow 2 = -2B \Rightarrow B = -1$

$\dfrac{2(r+3)}{r(r+1)(r+2)} = \dfrac{3}{r(r+1)} - \dfrac{1}{(r+1)(r+2)}$

දී ඇති ශ්‍රේණියේ r වන පදය U_r නම්,

$U_r = \dfrac{[8 + (r-1)2]}{[1(r-1)][2 + (r-1)][3 + (r-1)]} \left(\dfrac{1}{3}\right)^r$

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$$U_r = \frac{2r+6}{r(r+1)(r+2)}\left(\frac{1}{3}\right)^r = \frac{2(r+3)}{r(r+1)(r+2)}\left(\frac{1}{3}\right)^r$$

ඉහත ඔප්පු කළ සම්බන්ධය $\left(\frac{1}{3}\right)^r$ මගින් ගුණ කරමු.

$$\frac{2(r+3)}{r(r+1)(r+2)}\left(\frac{1}{3}\right)^r = \frac{3}{r(r+1)}\left(\frac{1}{3}\right)^r - \frac{1}{(r+1)(r+2)}\left(\frac{1}{3}\right)^r$$

$$U_r = \frac{1}{r(r+1)}\left(\frac{1}{3}\right)^{r-1} \frac{1}{(r+1)(r+2)}\left(\frac{1}{3}\right)^r \quad \therefore U_r = f(r-1) - f(r)$$

මෙහි $f(r) = \frac{1}{(r+1)(r+2)}\left(\frac{1}{3}\right)^r$ වේ.

$$\sum_{r=1}^{n} U_r = \sum_{r=1}^{n} f(r-1) - \sum_{r=1}^{n} f(r)$$

$$= \left\{\sum_{r=1}^{n} f(r) + f(0) - f(n)\right\} - \sum_{r=1}^{n} f(r)$$

$$= \frac{1}{1.2} - \frac{1}{(n+1)(n+2)}\left(\frac{1}{3}\right)^n$$

$$\sum_{r=1}^{n} U_r = f(0) - f(n)$$

$$= \frac{1}{12} - \frac{1}{(n+1)(n+2)}\left(\frac{1}{3}\right)^n //$$

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22. $$\frac{r+4}{r(r+1)(r+2)} = \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$$

$$r+4 = A(r+2) + Br$$

$r=0$; $4 = 2A$
$A = 2$ //

$r=-2$; $2 = -2B$
$B = -1$ //

$$\frac{r+4}{r(r+1)(r+2)} = \frac{2}{r(r+1)} - \frac{1}{(r+1)(r+2)}$$

$$U_r = \frac{r+4}{2^r r(r+1)(r+2)} = \frac{1}{2^r}\left\{\frac{2}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right\}$$

$$= \frac{1}{2^{r-1}\, r(r+1)} - \frac{1}{2^{r}\,(r+1)(r+2)}$$

$f(r) = \dfrac{1}{2^{r}\,(r+1)(r+2)}$ ලෙස ගත්විට, $U_r = f(r-1) - f(r)$

$r = 1\,;\quad U_1 = f(0) - f(1)$

$r = 2\,;\quad U_2 = f(1) - f(2)\,;$

$r = n-1\,;\quad U_{n-1}\; f(n-2) - f(n-1)$

$r = n\,;\quad U_n = f(n-1) - f(n)$

$$\sum_{r=1}^{n} U_r = f(0) - f(n) = \frac{1}{1 \times (1)\,(2)} - \frac{1}{2^{n}(n+1)\,(n+2)} = \frac{1}{2} - \frac{1}{2^{n}(n+1)\,(n+2)}$$

$$S_n = \sum_{r=1}^{n} \frac{r+4}{2^{r}\,(r)\,(r+1)(r+2)} = \frac{1}{2}\;\; \frac{1}{2^{n}\,(n+1)(n+2)}$$

$$S_\infty = \lim_{n \to \infty} \sum_{r=1}^{n} U_r = \lim_{n \to \infty} \frac{1}{2}\;\; \frac{1}{2^{n}\,(n+1)(n+2)}$$

$$= \frac{1}{2}$$

$\therefore\; S_1 \leq S_n \leq S_\infty$ වේ.

$$0 < \frac{1+4}{2^{1} \times 1 \times (1+1)\,(1+2)} \leq S_n \leq S_\infty = \frac{1}{2}$$

$0 < S_n < \dfrac{1}{2}$ වේ.

---

23. $U_r = \dfrac{r}{1 + r^{2} + r^{4}}$

$$U_r = \frac{r}{(1+r^{2})^{2} - r^{2}} = \frac{1}{2}\;\frac{2r}{\{(1+r^{2}) - r\}\{(1+r^{2}) + r\}}$$

$$= \frac{1}{2}\;\frac{(1+r+r^{2}) - (1-r+r^{2})}{(1+r+r^{2})\,(1-r+r^{2})}$$

$$= \frac{1}{2}\;\frac{1}{(1-r+r^{2})} - \frac{1}{(1+r+r^{2})}$$

$$U_r = \frac{1}{2}\left\{\frac{1}{\left(r-\frac{1}{2}\right)^2+\frac{3}{4}} \quad \frac{1}{\left(r+\frac{1}{2}\right)^2+\frac{3}{4}}\right\}$$

$$U_r = \frac{1}{2}\{f(r)-f(r+1)\}$$ මෙහි $f(r) = \frac{1}{1-r+r^2}$ වේ.

$$r=1\,;\ U_1 = \frac{1}{2}\{f(1)-f(2)\}$$

$$r=2\,;\ U_2 = \frac{1}{2}\{f(2)-f(3)\}\vdots$$

$$r=n-1 : U_{n-1} = \frac{1}{2}\{f(n-1)-f(n)\}$$

$$r=n\,;\ U_n = \frac{1}{2}\{f(n)-f(n+1)\}$$

$$\sum_{r=1}^{n} U_r = \frac{1}{2}\{f(1)-f(n+1)\}$$

$$= \frac{1}{2}\left\{\frac{1}{1-1+1^2} - \frac{1}{1-(n+1)+(n+1)^2}\right\}$$

$$= \frac{1}{2}\left\{\frac{1-(n+1)+(n+1)^2-1}{1-n-1+(n+1)^2}\right\}$$

$$= \frac{1}{2}\,\frac{(n+1)\{-1+(n+1)\}}{n^2+2n+1-n}$$

$$= \frac{n(n+1)}{2(n^2+n+1)}$$

විසඳු ගැටළු 03

ගැටළු

24. $1^2+2^2+3^2+\dots\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$ බව සාධනය කරන්න.

එනයින්, $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \dots\dots\dots$ ශ්‍රේණියේ පළමු පද n හි වෛක්‍යය සොයන්න. මෙම ශ්‍රේණිය අභිසාරී වේද? ඔබේ පිළිතුර සනාථ කරන්න.

25. n ධන නිඛිල සඳහා $U_n = \frac{1}{6} n(n+1)(n+2)$ වේ. $\frac{1}{U_n} = V_n - V_{n+1}$ වන පරිදි V_n සොයන්න. එනයින් $\sum_{r=1}^{n} \frac{1}{U_r} = \frac{3}{2} - \frac{3}{(n+1)(n+3)}$ බව පෙන්වන්න. $\sum_{r=1}^{\infty} \frac{1}{U_r}$ හි අගය අපෝහනය කරන්න.

26. $V_r - V_{r-1} = 2r \,.\, (r \geq 2)$ හා $V_1 = 1$ නම් $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$ භාවිතයෙන් හෝ අන්ක්‍රමයකින් හෝ $V_n = n^2 + n - 1$ බව පෙන්වන්න.

$U_r = \frac{V_r}{(r+2)!}$ නම් $f(r) - f(r+1) = U_r$ වන $f(r)$ ශ්‍රිතයක් සොයන්න. එනයින් $\sum U_r$ අගයන්න.

පිළිතුරු

24. $\sum_{r=1}^{n} = \frac{n}{6}(n+1)(2n+1)$ බව පෙන්වීමට ගණිත අභ්‍යුහන මූලධර්මය භාවිත කල හැකිය.

n = 1 විට ප්‍රතිඵලය සත්‍ය බව පෙන්වීම.

n = 1 විට ව.පැ. $= 1^2 = 1$. ද.පැ. $\frac{1}{6}(2+1)(1+1) = 1$

$\therefore$ n = 1 විට ව.පැ. = ද.පැ. වේ.

$\therefore$ n = 1 විට ප්‍රතිඵලය සත්‍ය වේ.

$\therefore$ n = p විට ප්‍රතිඵලය සත්‍ය යැයි උපකල්පනය කරමු.

එවිට, $1^2 + 2^2 + 3^2 + \ldots\ldots + p^2 = \frac{p}{6}(p+1)(2p+1)$

n = p + 1 ට ප්‍රතිඵලය සත්‍ය බව පෙන්වීම.

$$1^2 + 2^2 + \ldots\ldots + p^2 + (p+1)^2 = \frac{p}{6}(p+1)(2p+1) + (p+1)^2$$

$$= \frac{(p+1)\,p\,(2p+1) + 6\,(p+1)}{6}$$

$$= \frac{(p+1)(2p+3)(p+2)}{6}$$

$$= \frac{\overline{(p+1)}}{6}(\overline{p+1}+1)\,[2\,(\overline{p+1})+1]$$

∴ n = p විට ප්‍රතිඵලය සත්‍ය නම් n = p + 1 ට ප්‍රතිඵලය සත්‍ය වේ.

∴ ගණිත අභ්‍යුහන මූලධර්මයට අනුව සියලු $n \in Z^+$ සඳහා ප්‍රතිඵලය සත්‍ය වේ.

$\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots\ldots\ldots$ n වන පදය

$$T_n = \frac{3+2(n-1)}{1^2+2^2+\ldots\ldots+n^2} = \frac{2n+1}{n(n+1)(2n+1)/6}$$

$$= \frac{6}{n(n+1)} = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$T_n = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

n = 1 විට $T_1 = 6\left[1 - \frac{1}{2}\right]$

n = 2 විට $T_2 = 6\left[\frac{1}{2} - \frac{1}{3}\right]$

⋮ ⋮

n = n විට $T_n = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$

$$S_n = \sum_{r=1}^{n} T_r = 6\left[1 - \frac{1}{n+1}\right]$$

$$\therefore S_n = \frac{6n}{n+1}$$

$n \to \infty$ විට $S_n \to 6$ ∴ ශ්‍රේණිය අභිසාරී වේ.

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25. $U_n = \frac{1}{6} n(n+1)(n+2)$

$$\frac{1}{U_r} = \frac{6}{r(r+1)(r+2)}$$

$$= \frac{3}{r(r+1)} - \frac{3}{(r+1)(r+2)}$$

$V_r = \frac{3}{r(r+1)}$ ලෙස ගනිමු.

$r = 1$ විට $\frac{1}{U_1} = v_1 - v_2$

$r = 2$ විට $\frac{1}{U_2} = v_2 - v_3$

....

....

$r = n$ විට $\frac{1}{U_2} = v_1 - v_{n+1}$

$$\sum_{r=1}^{n} \frac{1}{U_r} = V_1 - V_{n+1}$$

$$= \frac{3}{2} - \frac{3}{(n+1)(n+3)}$$

$$\sum_{r=1}^{\alpha} \frac{1}{U_r} = \lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{U_r}$$

$$\lim_{n \to \alpha} \left(\frac{3}{2} - \frac{3}{(n+1)(n+2)} \right)$$

$$= \frac{3}{2}$$

26.

$$V_r - V_{r-1} = 2r$$

$r=2$ $\quad V_2 - V_1 = 2.2$

$r=3$ $\quad V_3 - V_2 = 2.3$

$r=4$ $\quad V_4 - V_3 = 2.3$

....

$r=n$ $\quad V_n - V_{n-1} = 2.n$

$$V_n - V_1 = 2\,[2+3+ \ldots\ldots\ n]$$

$$V_n - 1 = 2 \sum_{r=1}^{n} r - 2$$

$$V_n = 2.\frac{n}{2}(n+1) - 1$$

$$V_n = n^2 + n - 1$$

$$V_n = \frac{Vr}{(r-2)!} = \frac{r^2+r-1}{(r+2)!}$$

$$= \frac{r(r+2)-(r+1)}{(r+2)!}$$

$$= \frac{r}{(r+1)!} - \frac{(r+1)}{(r+2)!}$$

$f(r) = \dfrac{r}{(r+1)!}$ වේ.

$$U_r = f(r) - f(r+1)$$

$$U_1 = f(1) - f(2)$$

$$U_2 = f(2) - f(3)$$

$$U_2 = f(3) - f(4)$$

....

$$\sum_{r=1}^{1} U_r = f(1) - f(n+1)$$

$$= \frac{1}{2} - \frac{(n+1)}{(n+2)!}$$

විසඳු ගැටළු 04

ගැටළු

27. $\dfrac{1}{1!} + \dfrac{1+3}{2!} + \dfrac{1+3+5}{3!} + \ldots\ldots\ldots$ ශ්‍රේණියේ r වන පදය $U_r = \dfrac{r^2}{r!}$ බව පෙන්වන්න.

$e = \displaystyle\sum_{r=0}^{\infty} \frac{1}{r!}$ ලෙස ගනිමින් $\displaystyle\sum_{r=1}^{\infty} U_r = 2e$ බව පෙන්වන්න.

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28. $\frac{4}{1!} + \frac{11}{2!} + \frac{22}{3!} + \frac{37}{4!} + \frac{56}{5!} + ...$ ශ්‍රේණියේ r වන පදය U_r , $U_r = \frac{2.r^2 + r + 1}{r!}$

මගින් ගෙන දේ. $\sum_{r=1}^{\infty} U_r = 6e - 1$ බව පෙන්වන්න. මෙහි $e = 1 + \sum_{r=1}^{\infty} \frac{1}{r!}$ වේ.

29. $\sin^3 x + \frac{1}{3} \sin^3 3x + \frac{1}{3^2} \sin^3 9x + ...$ ශ්‍රේණියේ r වන පදය $U_r = \frac{1}{3^{r-1}} . \sin^3 3^{r-1} x$

බව දී ඇත. $U_r = \frac{1}{4} [f(r-1) - f(r)]$ වන පරිදි $f(r)$ සොයන්න.

එනයින් $\sum_{r=1}^{n} U_r = \frac{3}{4} \sin x - \frac{1}{4.3^{n-1}} \sin 3^n x$ බව පෙන්වන්න.

පිළිතුරු

27. $U_r = \frac{1 + 3 + 5 + ... + (2r-1)}{r!} = \frac{\frac{r}{2} . \{1 + (2r-1)\}}{r!}$

$$= \frac{\frac{r}{2} \times 2r}{r!} = \frac{r^2}{r!}$$

$$U_r = \frac{r^2}{r!} = \frac{r}{(r-1)!} = \frac{(r-1)+1}{(r-1)!} = \frac{(r-1)}{(r-1)!} + \frac{r}{(r-1)!}$$

$U_r = \frac{1}{(r-2)!} + \frac{r}{(r-1)!}$, $r \geq 2$ වේ.

$$\sum_{r=2}^{\infty} U_r = \sum_{r=2}^{\infty} \left(\frac{1}{(r-2)!} + \frac{1}{(r-1)!}\right) = \sum_{r=2}^{\infty} \frac{1}{(r-2)!} + \sum_{r=2}^{\infty} \frac{r}{(r-1)!}$$

$$\sum_{r=2}^{\infty} U_r + U_1 = e + \left(e - \frac{1}{0!}\right) + U_1$$

$$= e + e - 1 + \frac{1}{1!}$$

$$= 2e - 1 + 1 = 2e$$

28. $\dfrac{2.r^2+r+1}{r!} = \dfrac{A}{(r-2)!} + \dfrac{B}{(r-1)!} + \dfrac{C}{r\,!}$ වන පරිදි A, B, C සොයමු.

$2.r^2+r+1 = Ar\,(r-1)+B\,(r)+C$

$r=0$ විට $1=C$

$r=1$ විට $2+1+1=B+C$

$B = 3$

r^2 සංගුණකය, $2 = A$

$\therefore A=2\,,\; B=3\,,\; C=1$ //

$\dfrac{4}{1!}+\dfrac{11}{2!}+\dfrac{22}{3!}+\dfrac{37}{4!}+\dfrac{56}{5!}+$.......... ශ්‍රේණියේ r වන පදය $U_r = \dfrac{2.r^2+r+1}{r!}$

$$U_r = \frac{2}{(r-2)!}+\frac{3}{(r-1)!}+\frac{1}{(r)\,!}$$

$$\sum_{r=2}^{\infty} U_r = \sum_{r=2}^{\infty}\frac{1}{(r-2)!}+3\sum_{r=2}^{\infty}\frac{1}{(r-1)!}+\sum_{r=2}^{\infty}\frac{1}{(r)\,!} = 2\,(e)+3(e-1)+\left(e-1-\frac{1}{1!}\right)$$

$$\sum_{r=2}^{\infty} U_r = 6e-5$$

$$\sum_{r=1}^{\infty} U_r = 6e-5+U_1 = 6e-5+\frac{4}{1!}$$

$$\therefore \sum_{r=1}^{\infty} U_r = 6e-1$$ //

---

29. $\boxed{\sin 3\theta = 3\sin\theta - 4\sin^3\theta}$

$$U_r = \frac{1}{3^{r-1}}\sin^3 3^{r-1}x + \frac{1}{3^{r-1}}\left[\frac{3\sin^3 3^{r-1}x - \sin 3\times(3^{r-1}x)}{4}\right]$$

$$= \frac{1}{4}\left[\frac{1}{3^{r-2}}\,.\,\sin^3 3^{r-1}x - \frac{1}{3^{r-1}}\,.\,\sin^3 3^{r}x\right]$$

$$= \frac{1}{4}\,[f(r-1)-f(r)]$$

$\therefore f(r) = \frac{1}{3^{r-1}} \cdot \sin 3^r x$ වේ.

$r = 1 \quad U_1 = \frac{1}{4}[f(0) - f(1)]$

$r = 2 \quad U_2 = \frac{1}{4}[f(1) - f(2)]$

$\vdots$

$r = n-1 \quad U_{n-1} = \frac{1}{4}$

$r = n \quad U_n = \frac{1}{4}[f(n-1) - f(n)]$

$$\sum_{r=1}^{n} U_r = \frac{1}{4}[f(0) - f(n)]$$

$$= \frac{1}{4}\left[\frac{1}{3^{0-1}} \sin 3^0 x - \frac{1}{3^{n-1}} \cdot \sin 3^n x\right]$$

$$= \frac{3}{4} \sin x - \frac{1}{4.3^{n-1}} \sin 3^n x$$